II. AN EXAMINATION OF VARIOUS THEORETICAL APPROACHES CONSISTENT WITH ECONOMIC THEORY THAT MATHEMATICALLY REPRESENT THE "MARGINAL" INVESTMENT OF NEW DEMAND, AND HOW THEY CONVERGE TO WHAT IS COMMONLY KNOWN AS A "CAPACITY COST." (Continued)

b) Mathematical Proofs, Proof I. (Continued)

Conclusion of Proof:

Now, assume that the initial demand (ℓ_0) at $t=0$ is uniformly distributed over the interval $0 \leq \ell_0 \leq q$.

Then, $LRMC = \frac{1}{q} \cdot \int_0^q LRMC \, d\ell_0$

$$= \frac{\delta\beta}{d} \cdot \frac{e^{\frac{-\delta q}{d}}}{1-e^{\frac{-\delta q}{d}}} \cdot \left(\frac{1}{q} \cdot \int_0^q e^{\frac{\delta \ell_0}{d}} \, d\ell_0 \right)$$

$$= \frac{\delta\beta}{d} \cdot \frac{e^{\frac{-\delta q}{d}}}{1-e^{\frac{-\delta q}{d}}} \cdot \frac{1}{q} \cdot \frac{d}{\delta} \cdot \left(e^{\frac{\delta q}{d}} - 1 \right)$$

$$= \frac{\beta}{q} \cdot \frac{e^{\frac{-\delta q}{d}} \cdot \left(e^{\frac{\delta q}{d}} - 1 \right)}{1-e^{\frac{-\delta q}{d}}}$$

$$= \frac{\beta}{q} \cdot \frac{1-e^{\frac{-\delta q}{d}}}{1-e^{\frac{-\delta q}{d}}}$$

$$= \frac{\beta}{q} \tag{10}$$

$$= \frac{\text{INVESTMENT}}{\text{CAPACITY}}$$

As can be seen from the equations (1) and (10), $LRMC$ = capacity cost.

b) Mathematical Proofs (Continued)

Proof II.

Definitions: (See Figure 5)

Let q = capacity of machine or equipment unit (CAP)
β = investment of machine or equipment unit (INV)
ℓ_o = initial demand at $t=0$
η = slope of demand
α = slope of new demand (with an increased demand)
i = interest rate
δ = $\ell n\,(1+i)$
CC = capacity cost
PVC = change in present value of investment
PVD = change in present value of demand
$LRMC$ = long run marginal investment

In Figure 4, it is assumed that an added demand is constant in relation to the baseline. That is, the slope of new demand is the same as that of the baseline. Here, we allow a demand growth rate. As can be seen in Figure 5, the slope of new demand is different from that of the baseline.

Hypothesis:

The capacity cost is defined to be:

$$\text{capacity cost } (CC) = \frac{\beta}{q} = \frac{INV}{CAP} \qquad (1)$$

$$\frac{PVC}{PVD} = MC \stackrel{?}{=} CC$$

b) Mathematical Proofs, Proof II. (Continued)

Detailed Equations:

As before, if we assume that an increase in demand at $t=0$ does not exceed the remaining capacity of the machine or equipment unit, we find that in general:

$\ell_j + (\alpha - \eta)\, t_j = jq$ and

$$t_j = \frac{jq - \ell_j}{\alpha - \eta}, \quad j = 1, 2, \cdots, \infty \tag{2}$$

To calculate the PVC, we first denote a gradient series, β, in terms of a uniform series, $\frac{\beta}{\delta}$, for the case of perpetuity. Then the PVC is determined as:

$$PVC = \sum_{j=1}^{\infty} \frac{\beta}{\delta}\, e^{-\delta t_j} \tag{3}$$

$$= \frac{\beta}{\delta} \cdot \sum_{j=1}^{\infty} e^{-\delta\left(\frac{jq - \ell_j}{\alpha - \eta}\right)}$$

$$= \frac{\beta}{\delta} \cdot e^{\frac{\delta \ell_j}{\alpha - \eta}} \cdot \sum_{j=1}^{\infty} e^{\frac{-jq\delta}{\alpha - \eta}}$$

$$= \frac{\beta}{\delta} \cdot e^{\frac{\delta \ell_j}{\alpha - \eta}} \cdot \frac{e^{\frac{-q\delta}{\alpha - \eta}}}{1 - e^{\frac{-q\delta}{\alpha - \eta}}} \tag{4}$$

To calculate the PVD, a gradient series in change in demand $\alpha - \eta$ is expressed in terms of a uniform series $\frac{\alpha - \eta}{\delta}$. Then, the PVD is determined as:

$$PVD = \int_0^{\infty} \frac{\alpha - n}{\delta}\, e^{-\delta t}\, dt = \frac{\alpha - n}{\delta^2} \tag{5}$$

b) Mathematical Proofs, Proof II. (Continued)

Detailed Equations: (Continued)

From equations (4) and (5),

$$LRMC = \frac{PVC}{PVD}$$

$$= \frac{\beta\delta}{\alpha-\eta} \cdot \frac{e^{\frac{-q\delta}{\alpha-\eta}}}{1-e^{\frac{-q\delta}{\alpha-\eta}}} \cdot e^{\frac{\delta\ell_o}{\alpha-\eta}} \tag{6}$$

Conclusion of Proof:

Assuming ℓ_o is uniformly distributed over the interval $0 \leq \ell_o \leq q$,

$$LRMC = \frac{1}{q} \cdot \int_0^q LRMC \, d\ell_o$$

$$= \frac{\beta\delta}{\alpha-\eta} \cdot \frac{e^{\frac{-q\delta}{\alpha-\eta}}}{1-e^{\frac{-q\delta}{\alpha-\eta}}} \cdot \left(\frac{1}{q} \cdot \int_0^q e^{\frac{\delta\ell_o}{\alpha-\eta}} \, d\ell_o \right)$$

$$= \frac{\beta\delta}{\alpha-\eta} \cdot \frac{e^{\frac{-q\delta}{\alpha-\eta}}}{1-e^{\frac{-q\delta}{\alpha-\eta}}} \cdot \frac{\alpha-\eta}{q\delta} \cdot \left(e^{\frac{\delta q}{\alpha-\eta}} - 1 \right)$$

$$= \frac{\beta}{q} \cdot \frac{1-e^{\frac{-q\delta}{\alpha-\eta}}}{1-e^{\frac{-q\delta}{\alpha-\eta}}}$$

$$= \frac{\beta}{q} \tag{7}$$

$$= \frac{INV}{CAP}$$

b) Mathematical Proofs, Proof II. (Continued)

Conclusion of Proof: (Continued)

Therefore, from equations (1) and (7),

$$LRMC = \frac{\text{INVESTMENT}}{\text{CAPACITY}}$$

$$= \text{CAPACITY COST } (CC)$$

II. AN EXAMINATION OF VARIOUS THEORETICAL APPROACHES CONSISTENT WITH ECONOMIC THEORY THAT MATHEMATICALLY REPRESENT THE "MARGINAL" INVESTMENT OF NEW DEMAND, AND HOW THEY CONVERGE TO WHAT IS COMMONLY KNOWN AS A "CAPACITY COST." (Continued)

b) Mathematical Proofs (Continued)

Proof III.

Definitions:

In this section, we show that the $LRMC$ proposed by U S WEST Communication (SCM Group, R. W. Foster and R. M. Bowman, April, 1989) is also equivalent to the capacity cost with arrival of demand given an equal probability. For a reference, their $LRMC$ is reproduced exactly as in their paper (see Figure 6).

$$LRMC = \begin{cases} \dfrac{Io}{Qo} \cdot \dfrac{(1+R)^{To}}{F} \cdot \dfrac{(1+R)^{FN}-1}{(1+R)^{N}-1}, & 0 \leq To \leq N(1-F) \\ \dfrac{Io}{Qo}\left[\dfrac{1}{F} + \dfrac{(1+R)^{To}}{F} \cdot \left(\dfrac{(1+R)^{FN}-1}{(1+R)^{N}-1} - \dfrac{(1+R)^{FN}}{(1+R)^{N}}\right)\right], & N(1-F) < To \leq N \end{cases} \tag{1}$$

, where:

Qo = capacity of machine
Io = investment of machine
To = time at which new service is offered
F = demand for new services
N = years until exhaustion of a new capacity addition
R = interest rate (cost of money)
δ = $\ell n(1+R)$
MC = marginal cost

Hypothesis:

The capacity cost is determined as:

$$\text{Capacity Cost (CC)} = \frac{Io}{Qo} = \frac{\text{Investment}}{\text{Capacity}} \tag{2}$$

$$MC \stackrel{?}{=} CC$$

II. AN EXAMINATION OF VARIOUS THEORETICAL APPROACHES CONSISTENT WITH ECONOMIC THEORY THAT MATHEMATICALLY REPRESENT THE "MARGINAL" INVESTMENT OF NEW DEMAND, AND HOW THEY CONVERGE TO WHAT IS COMMONLY KNOWN AS A "CAPACITY COST." (Continued)

b) Mathematical Proofs, Proof III. (Continued)

Detailed Equations:

Equation (1) can be expressed as:

$$LRMC = \begin{cases} \dfrac{Io}{Qo} \cdot \dfrac{e^{To\delta}}{F} \cdot \dfrac{e^{FN\delta}-1}{e^{N\delta}-1}, & 0 \le To \le N(1-F) \\[2ex] \dfrac{Io}{Qo}\left[\dfrac{1}{F} - \dfrac{e^{To\delta}}{F} \cdot \left(\dfrac{e^{FN\delta}-1}{e^{N\delta}-1} - \dfrac{e^{FN\delta}}{e^{N\delta}}\right)\right], & N(1-F) < To \le N \end{cases} \tag{3}$$

Conclusion of Proof:

Now assume that To (time at which new service is offered) is uniformly distributed over the interval $0 \le To \le N$.

Then,

$$LRMC = \frac{1}{N} \cdot \int_0^N LRMC \; dTo$$

$$= \frac{1}{N}\left[\int_0^{N(1-F)} \frac{Io}{QoF} \cdot \frac{e^{FN\delta}-1}{e^{N\delta}-1}\, e^{To\delta}\, dTo \right.$$

$$\left. + \int_{N(1-F)}^{N}\left(\frac{Io}{QoF} + \frac{Io}{QoF} \cdot \frac{e^{FN\delta}-1}{e^{N\delta}-1}\, e^{To\delta} - \frac{Io}{QoF}\,\frac{e^{FN\delta}}{e^{N\delta}}\, e^{To\delta}\right) dIo\right]$$

$$= \frac{Io}{NQoF}\left[\frac{1}{\delta} \cdot \frac{e^{FN\delta}-1}{e^{N\delta}-1} \cdot \left(e^{N(1-F)\delta}-1\right) + NF\right.$$

$$\left. + \frac{1}{\delta} \cdot \frac{e^{FN\delta}-1}{e^{N\delta}-1} \cdot \left(e^{N\delta} - e^{N(1-F)\delta}\right) - \frac{1}{\delta}\,\frac{e^{FN\delta}}{e^{N\delta}} \cdot \left(e^{N\delta} - e^{N(1-F)\delta}\right)\right]$$

$$= \frac{Io}{NQoF}\left[\frac{e^{N\delta} - e^{FN\delta} - e^{N(1-F)\delta} + 1}{\delta(e^{N\delta}-1)} + NF\right.$$

$$\left. + \frac{e^{N(1+F)\delta} - e^{N\delta} - e^{N\delta} + e^{N(1-F)\delta}}{\delta(e^{N\delta}-1)} - \frac{e^{N(1+F)\delta} - e^{N\delta}}{\delta e^{N\delta}}\right]$$

II. AN EXAMINATION OF VARIOUS THEORETICAL APPROACHES CONSISTENT WITH ECONOMIC THEORY THAT MATHEMATICALLY REPRESENT THE "MARGINAL" INVESTMENT OF NEW DEMAND, AND HOW THEY CONVERGE TO WHAT IS COMMONLY KNOWN AS A "CAPACITY COST." (Continued)

b) Mathematical Proofs, Proof III. (Continued)

Conclusion of Proof: (Continued)

$$LRMC = \frac{Io}{NQoF}\left[NF + \frac{e^{NF\delta}(e^{N\delta}-1)-(e^{N\delta}-1)}{\delta(e^{N\varepsilon}-1)} - \frac{e^{N\delta}(e^{NF\delta}-1)}{\delta e^{N\delta}} \right]$$

$$= \frac{Io}{NQoF}\left[NF - \frac{e^{NF\delta}-1}{\delta} - \frac{e^{NF\varepsilon}-1}{\delta} \right]$$

$$= \frac{Io}{NQoF}\left[NF \right]$$

$$= \frac{Io}{Qo} \qquad (4)$$

$$= \frac{\text{INVESTMENT}}{\text{CAPACITY}}$$

From equations (1) and (4), $LRMC$ = Capacity Cost.

c) Summary

The above three proofs all show that the capacity cost concept is a proper substitution for the more complex form of the LRMC. In the proofs, several steps were left out for brevity. Anyone who would like more detailed information regarding the proofs can write to: Victor Schmid-Bielenberg, Bellcore, Room LCC 2E233, 290 West Mt. Pleasant Ave., Livingston, N.J. 07039.

III. AN EXPLANATION OF HOW THE CAPACITY COST APPROACH IS APPLIED TO MAJOR INVESTMENT FUNCTIONS OF SCIS AND HOW SPECIAL SITUATIONS ARE HANDLED.

a) CAPACITY COST CONCEPT APPLICATION

Figure 7 shows a generic SCIS Model Office printout, listing all of the required investment primitives or building blocks which represent the minimum set required for determining any feature or service in a switch.

The following of these investment primitives are purchased as investment "lumps" over time, each providing an equal capacity increment, and therefore the capacity cost formula is applicable to them.

1) The main processor community, including maintenance and test equipment, spares, breakage, and any equipment that is purchased once, only as a function of a new switch entity required because its limiting resource has been exhausted for the previous unit. The most limiting resource for this is generally usable milliseconds or some other measure of Central Processing Unit (CPU) time utilized. Its driver, therefore is milliseconds, and its Investment per millisecond, based on the capacity cost concept is simply:

Investment per Busy Hour millisecond =

Getting Started Investment / maximum available milliseconds

where maximum available milliseconds =

absolute available milliseconds * allowed engineering fill=
60 mins * 60 secs per min * 1000 msecs per sec * vendor recommended fill factor for call processing

If the main processor never exhausts, the Long Run Marginal Investment for additional processing units (milliseconds) is zero.

2) The distributed processing community, including associated equipment, is purchased over time in units with fixed capacity, but it may be limited by 2 different resources, milliseconds and physical terminations. For this reason, the capacity cost concept is applied together with a remainder of spare capacity which is assigned to the terminations it serves (this special case is described in III. c.)

Investment per BH millisecond =
Distributed Processor Invest./max. available milliseconds

This is sometimes converted to an investment per equivalent POTS call.

III. AN EXPLANATION OF HOW THE CAPACITY COST APPROACH IS APPLIED TO MAJOR INVESTMENT FUNCTIONS OF SCIS AND HOW SPECIAL SITUATIONS ARE HANDLED. (CONTINUED)

a) CAPACITY COST CONCEPT APPLICATION (CONTINUED)

3) The Line Termination Investment is based on equipment provided on a one for one basis and is called non traffic sensitive equipment. This investment primitive is based on the capacity cost concept, since multiple terminations are accommodated by the same piece of equipment with a fixed capacity. Other parts are truly provided one for one, such as line cards and are, therefore, a capacity cost in its simplest form (i.e. one investment provides one capacity). Excess CCS and distributed processor capacity is also included. (See III. c.)

4) The line termination BH investment per CCS is determined using the capacity cost concept, since line peripherals have fixed CCS capacities by type of line and concentration ratio. Therefore, within a line type and concentration ratio, the line peripheral and associated equipment requires an investment (lumpy) and provides a fixed CCS capacity at a specified grade of service. Therefore,

Line Termination BH Investment per CCS =

Investment for Line Peripheral by type and conc. ratio / CCS capacity of unit at desired grade of service

This equipment unit also has a dual load and termination limit, and, therefore, the capacity cost concept is applied to this primitive, with the remainder of spare engineered capacity assigned to the investment per termination. (This will be further discussed in part III. c.).

5) The trunk termination investment per CCS is computed using the capacity cost concept for those items that share common trunk peripheral equipment. Specific trunk circuit equipment is added to the termination investment, before the total investment is divided by the CCS capacity of the next trunk unit to be added. Sometimes it is necessary to keep these investments on a per trunk (equivalent talking channel) basis, because they are dedicated to a customer, or become part of a network model, that computes interoffice facilities, based on its own algorithms. For those cases, an approach as shown in part III. b. is more appropriate.

6) 7) 8) 9) & 10) Call Investments

Call investments, if required, are generally based on a capacity cost, except in rare cases, were non linearities still exist. These will be itemized in part III. b.

III. AN EXPLANATION OF HOW THE CAPACITY COST APPROACH IS APPLIED TO MAJOR INVESTMENT FUNCTIONS OF SCIS AND HOW SPECIAL SITUATIONS ARE HANDLED. (CONTINUED)

a) CAPACITY COST CONCEPT APPLICATION (CONTINUED)

11) Special Hardware Investments consist of special service circuits, such as 3-port bridges, etc. and are assigned to feature investments based on their usage. Since their investment represents the total loaded investment of one of these special circuits, it requires loading the investment with frame space, memory, cabling, etc. by using the capacity cost concept. When completed, this process converts the special hardware investment into an investment per CCS. The CCS capacity of the next added unit is used for this, which makes it a slight variation of the capacity cost concept, since the capacity cost concept assumes equal capacities for all added units. But, for units that are added frequently this is a good approximation (small lumps of investment and capacity), without having to use a modified capacity cost concept or the more general marginal investment equation, as is the case for 13) below.

12) Investment per byte. This is an ideal capacity cost application. The Investment for the next memory unit is simply divided by the capacity it provides.

13) SS7 Investment per octet. One must first understand that this investment is driven by link pairs required which in turn are a function of octets. The first link pair in a given switch, is much more expensive than the next and so on. The capacity provided by the first link pair could be different than that of subsequent link pairs. The investment lumps, although variable, are large, and provide capacity, also variable, for a long time. For this reason, no approximation, using a modified capacity cost concept is appropriate for this investment primitive or building block. It is more complex and is not shown for the sake of brevity. An alternative is to require input data, that forecasts demand for a long time out, so that the present worth of investment streams and changes; and the present worth for demand streams and changes can be computed to obtain the marginal unit investment based on the original definition of a marginal cost:

Investment per BH octet =

PW of changes in investment over time /
PW of changes in demand over time

This is cumbersome and requires extensive input data. A modified capacity cost approach is available that provides an equally practical and accurate alternative as the simple capacity cost concept used for other investment functions.

a) CAPACITY COST CONCEPT APPLICATION (CONTINUED)

14) and 15) ISDN access and data packet investments per packet per second by type in general follow the capacity cost concept, although, for some technologies additional considerations are required due to their capacity interdependences. Details will not be provided here.

b) NON LINEAR INVESTMENT FUNCTIONS

Although it was quite common to see non linear investment functions in analog switches, these are extremely rare in digital switches. The few non linear functions in digital switches are limited to infrequently used building blocks or investment primitives, such as, the incoming call investment (analog only) for DMS. Since most interoffice trunking is using digital facilities, the application of this term is rare indeed. Regardless of its infrequent use, the appropriate marginal investment is shown in Figure 7.

Non switch based investments, such as investments of interoffice facilities, are not a function of SCIS, but of other cost models, such as Bellcore's Network Cost Analysis Tool (NCAT). Interoffice Facilities benefit from economies of scale creating non linear situations. Therefore, an approach that stimulates the CCS load on a given trunking route can be used to determine changes in trunk quantities and investments. Through this method, any non linearities or poisson effects are captured and treated accordingly.

c) TREATMENT OF DUAL LIMITING EQUIPMENT ITEMS

Some equipment items may have two limiting capacities. An example of this might be a line peripheral. It generally provides a given grade of service at a specified CCS capacity. At the same time, it provides a limited termination capacity. By selecting a concentration ratio that best fits the office average CCS per line, one must make a long term selection, which of the two limiting drivers is most serious and what is the best way to serve lines on the average. Switches that are limited on CCS, have only one significant driver, namely CCS, and the whole investment of the line peripheral becomes a function of CCS. Or, simply, the capacity cost, based on CCS, for each CCS consumed.

c) TREATMENT OF DUAL LIMITING EQUIPMENT ITEMS (CONTINUED)

For switches that are not at nor are projected to reach this CCS capacity point, the capacity cost is used for the CCS actually consumed. However, each additional group of average usage lines brings additional spare capacity into the switching system. Therefore, it is necessary to compute a component called excess capacity investment per line, which is a fixed cost (as long as the actual or projected office average CCS per Line is below the capacity), incurred as a function of lines, and, therefore, it becomes part of the minimum investment per line. Distributed processor excess capacity investments are similarly assigned to lines and trunks, where applicable.

This is consistent with cost causation principles used elsewhere and is somewhat analogous to an engineered fill that is a continuous requirement; a phenomenon also discussed in the next paragraph.

d) ENGINEERED FILL FACTORS

A fill factor is defined as the absolute capacity of a unit (i.e. 100%) minus an administrative amount of spare required to either allow for unforeseen demand, for churn of service orders, for load balance, for peak load protection, or for overhead tasks. For example, it might be 95% or .95 for peripheral terminations, or it might be 40% or .40 for a central processor that requires 60% of its time for overhead functions. Since this reduction in available capacity is experienced on all future units purchased, its effect becomes a real cost that needs to be reflected in cost studies.

For example, a central processor has only 40% of its real time available for call processing. Since processing capacity is measured in milliseconds, its absolute capacity in an hour is 3,600,000 milliseconds of real time per hour. (60 minutes * 60 seconds * 1000 milliseconds per second). Since only 40% of this real time is available for call processing, its fill adjusted capacity is only 3.6 Million milliseconds * .40, or 1.44 Million milliseconds. The capacity cost is therefore:

Total investment next unit (getting started investment) / Fill adjusted capacity

or

Total investment next unit / 3,600,000 * Fill Factor

or

Total investment next unit / 3,600,000 * .4

d) ENGINEERED FILL FACTORS (CONTINUED)

Similarly for line peripherals at 95% fill, one would need to buy approximately 5 extra line terminations for every 100 lines. Since this is not an avoidable cost as more lines are added, it needs to be part of the marginal cost.

A generalized formula is therefore simply this:

Marginal unit Investment (using the capacity cost concept)=

Investment of next unit / (Capacity of next unit * Fill Factor)

IV. PRACTICAL SOLUTION FOR DETERMINING THE MARGINAL INVESTMENT OF VARIOUS FEATURES AND SERVICES USING SCIS.

a) GENERAL

After having developed the foundation for a practical way of developing investment building blocks or primitives (using the methods described earlier in this paper) with a minimum set of investment functions required in determining the marginal investment for any feature or service, we can now go back and look at the application of the SCIS model.

Figure 9 shows the SCIS process as viewed by the cost analyst. Local discount factors, traffic volumes, number of lines by type, number of trunks by trunk type, etc. are input into the model by the user. The Bellcore provided model office equation is now utilized to customize the cost primitives or building blocks of Figure 8 to local conditions. At the same time, more than one switch can be processed, if desired, to represent a weighted average of each building block or investment primitive for a tariff area, for example. The individuality of each switch can be preserved, however for network (point to point) cost studies, if desired.

The output of SCIS, are unit investments, that now represent the required cost primitives by investment function, customized to each user's specific input data, that can now be used for any application.

b) DETERMINING THE MARGINAL INVESTMENT FOR A SINGLE SWITCH BASED FEATURE ("ISLAND FEATURE")

Figure 10 shows how a simple feature like three way calling utilizes the investment primitives from the model office output that apply to the feature. Three way calling is considered an "island feature", because all the intelligence to make it work is resident in the local switch, unlike an intelligent network service.

The user inputs some feature specific data into SCIS (see Figure 11), vendor resource measurements (i.e. milliseconds required, memory required, etc.) are stored in tables for each feature, Bellcore's formulae are applied, and an output is produced that shows the marginal investment required for equipping one line with three way calling (see Figure 12). This is then converted into an annual or monthly cost through the process described in part I of this paper.

There are over 800 applications (features and services) in SCIS for various vendors of switches.

b) DETERMINING THE MARGINAL INVESTMENT FOR A SINGLE SWITCH BASED FEATURE ("ISLAND FEATURE") (CONTINUED)

To provide more details for one of the output lines, the following example is given:

The vendor of a switch measures that "x" milliseconds are required for three way calling (both calls). Normally two calls without three way calling require "y" milliseconds. The investment of the calls themselves become part of another cost study. Therefore only "x" - "y" = "z" milliseconds are consumed by the feature itself.

We know the investment per millisecond from the Model Office investment primitive. The cost analyst has given us inputs related to how often three way calling is used in the Busy Hour by an average line with the feature (Fig. 11). The formula for the first equation now simply becomes:

Investment per Line for three way calling (Bellcore Formula) =

Investment per BH millisec. (investment primitive from model)
* "z" millisecs. per feature use (from table, vendor provided)
* Use of Feature in the BH (input by cost analyst)

All investment functions are examined to determine whether they play a role in the feature under study, and a similar process is followed if they do. The sum of all the investments that are caused by the use of the feature, becomes the total feature investment, as shown in Fig. 12.

Other feature investments are developed following the same methodology.

c) DETERMINING THE MARGINAL INVESTMENT FOR INTELLIGENT NETWORK FEATURES AND SERVICES (SS7 BASED)

Intelligent Network (IN) services require some switch based investment functions and primitives generated by SCIS, and also require their own family of investment primitives, generated from investment functions that are specific to the SS7 network components STPs, SCPs, and Links). Details of these will not be covered here. Suffice it to say that another SCIS like model called Common Channel Signaling Cost Information System (CCSCIS) (see Fig. 13), using the same principles as SCIS, is used to generate investment primitives or building blocks used for IN services.

c) DETERMINING THE MARGINAL INVESTMENT FOR INTELLIGENT NETWORK FEATURES AND SERVICES (SS7 BASED) (CONTINUED)

Since building blocks are required from two or more different systems (i.e. SCIS, CCSCIS, etc.) a typical IN service marginal cost study cannot be completed in any one of these models. For this reason, investment primitives from these models are handed to another model called NC-PRISM (See Figure 13) (Network Cost - Program Ring For Integrating Software Modules).

NC-PRISM contains the vendor resource tables and Bellcore application's formulae for a specific IN service, and through user inputs, investments for the whole service (end to end) are produced. A typical IN service, incurs marginal investments at the originating switch, at the links to STPs, at the links between STPs, at the links to SCPs if required, at the links from the STPs to the terminating switch, and at the terminating switch, to use a simplified example. This can all be pieced together by NC-PRISM based on the local topology of switch types, STP types, SCP types, link types, etc.

d) DETERMINING MARGINAL INVESTMENTS FOR VOICE NETWORK SERVICES

Two other models are made available by Bellcore, for network voice services cost studies:

NCAT - The Network Cost Analysis Tool, that produces local, intralata toll, and interlata access marginal costs by class of service, by time of day, by length of haul for point to point, multipoint, small area, LATA, or statewide studies.

OACIS - The Operator Analysis Cost Information System that produces marginal costs for Operator differential over dial requirements for various operator services or Alternate Billed Services (ABS) calls.

Both of these models benefit from the investment primitives that are generated by SCIS and CCSCIS and, together with local inputs for other investments and expenses, produce outputs that are consistent with the marginal cost philosophy discussed previously.

e) SUMMARY

This paper has shown a harmonious approach in providing marginal investments through SCIS and related Bellcore models for any switch or network based service. One does not need tedious mathematics with massive data requirements to obtain more than satisfactory results. Results that accurately represent marginal costs as defined in widely accepted articles of economic theory and principles.

By using a consistent approach and standards as defined in this paper, marginal cost concepts in the theoretical realm of economic theory are brought practically into the real world of daily applications as encountered by the cost analyst. These applications can be performed with the simplicity of the capacity cost concept, without sacrificing any credibility or reliability. These applications and concepts are relevant to the problem at hand, with relevant solutions through the use of investment building blocks or primitives. These applications and concepts are reasonable because they represent an approach that is simple, verifiable, consistent, and practical.

Generic Switch Functions - Any Switch

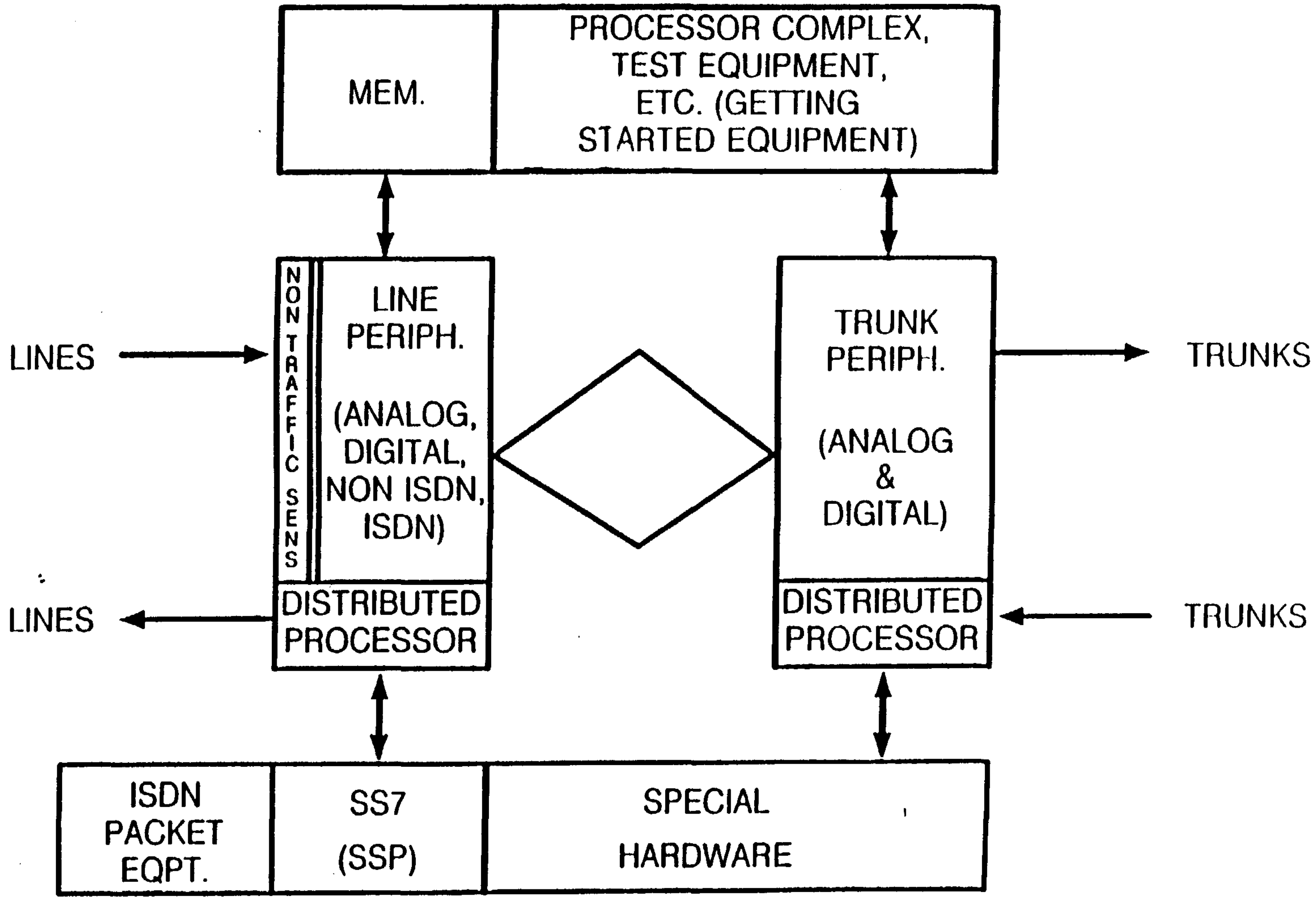


FIGURE 1

LCC90 BVS823.001

Generic Investment Functions - Any Switch

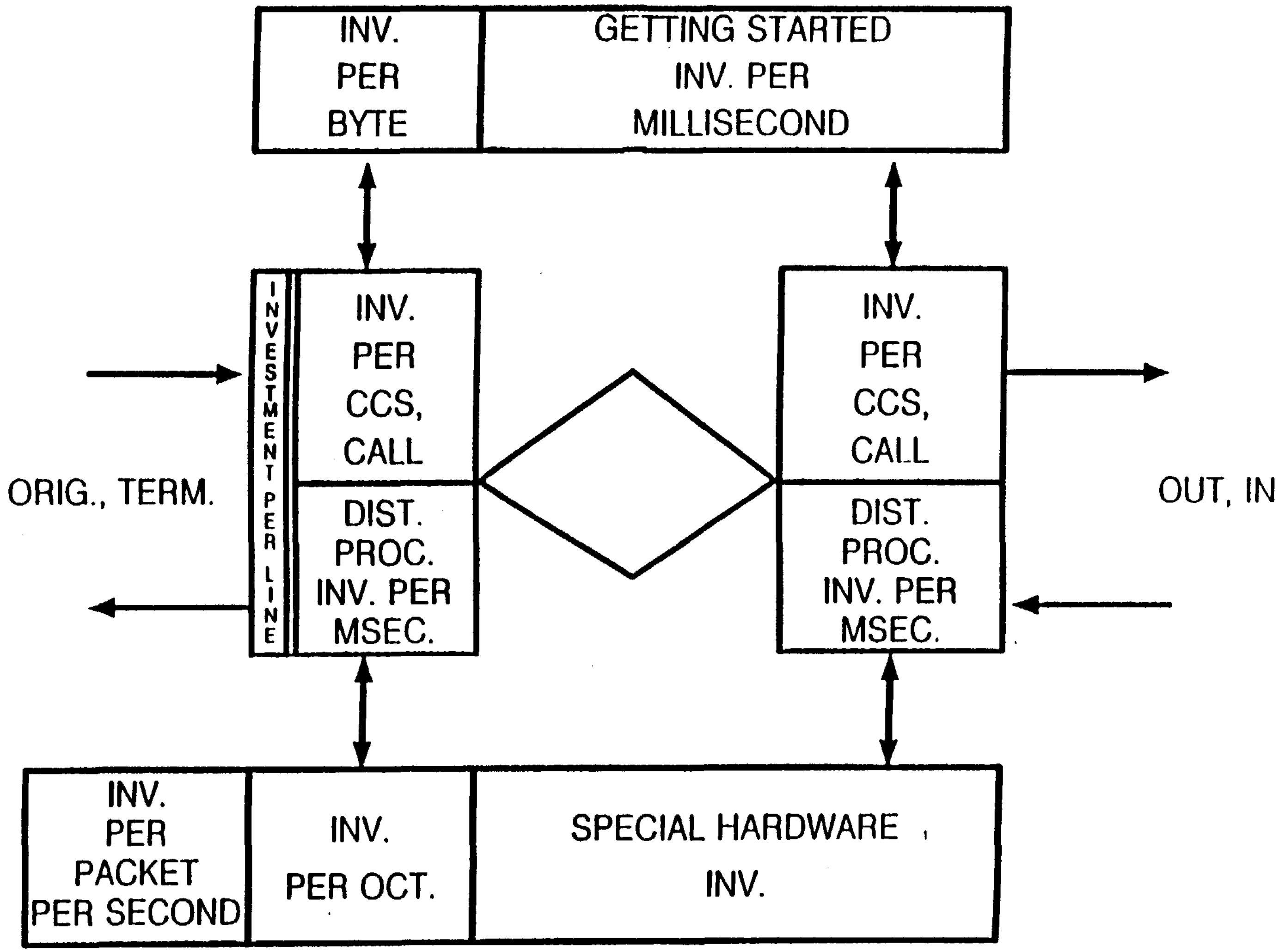
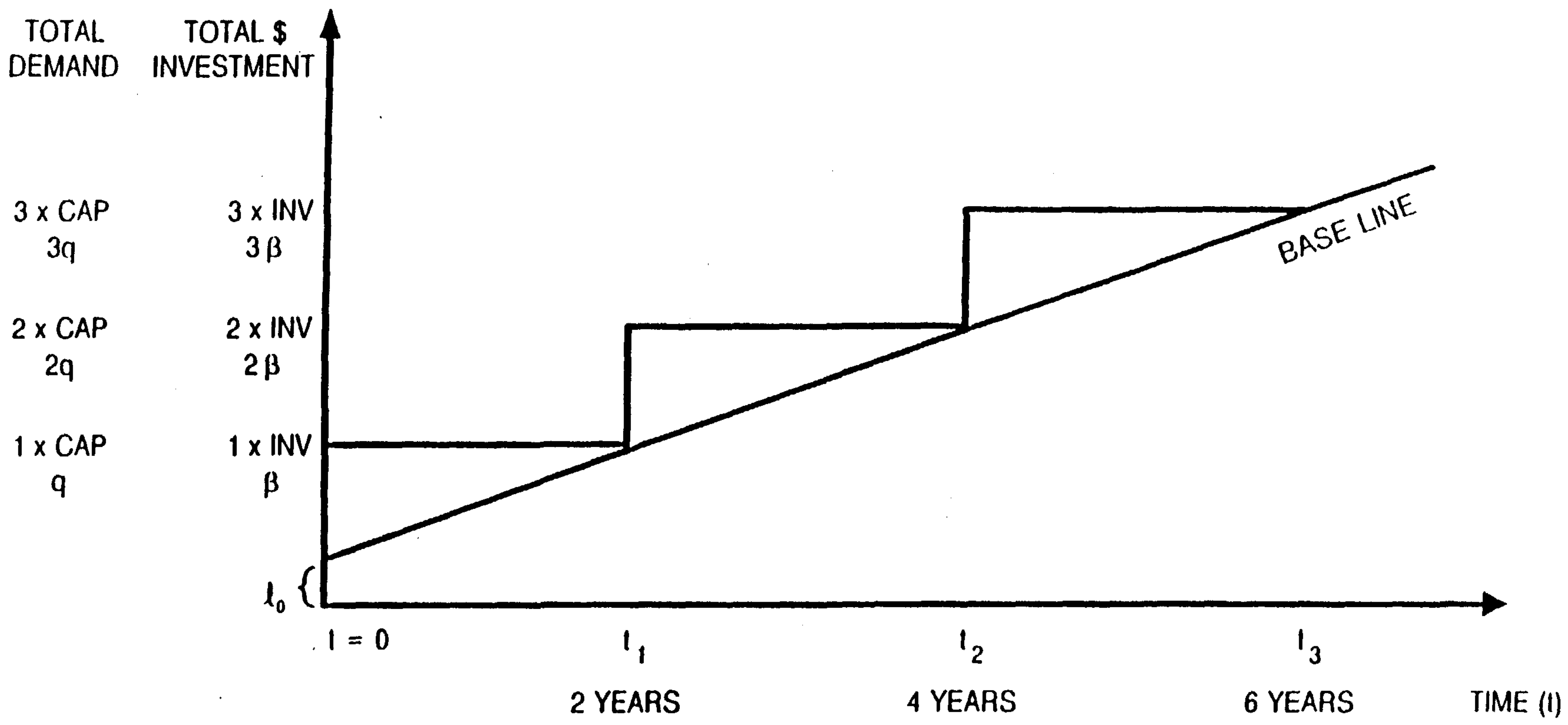


FIGURE 2

LCC90 BVS823.002

General Case



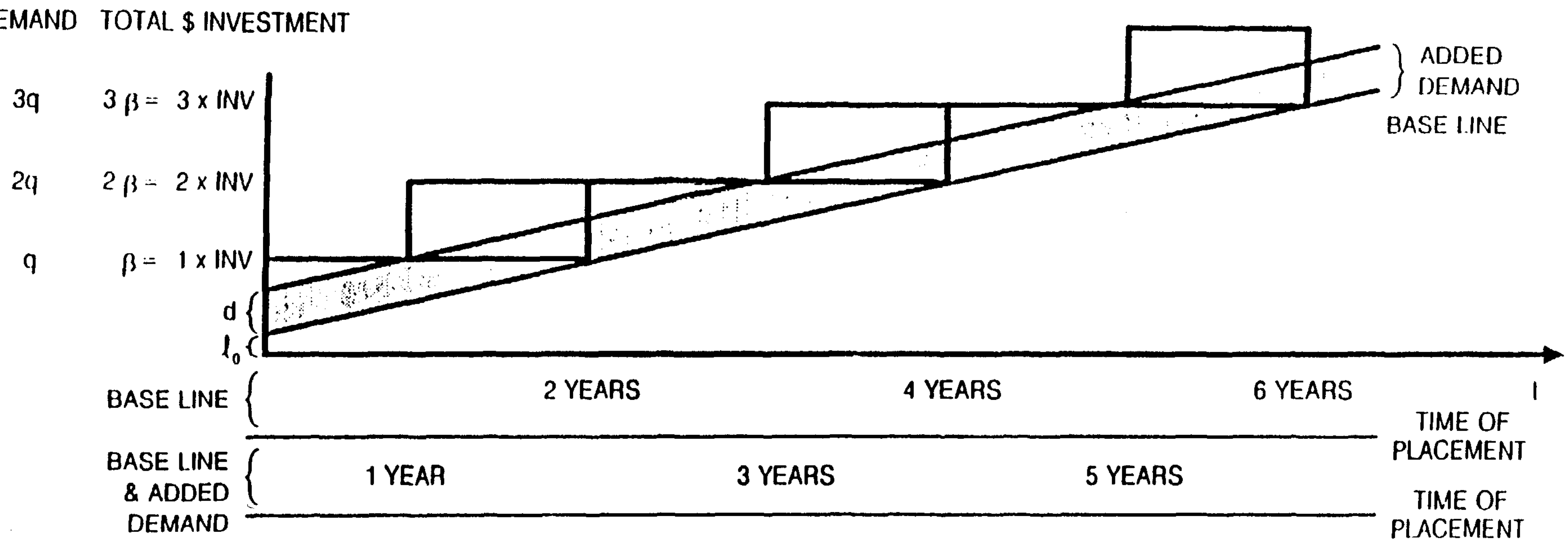
β = INV = INVESTMENT FOR 1 EQUIPMENT UNIT

q = CAP = CAPACITY OF 1 EQUIPMENT UNIT

FIGURE 3

LCC90 BVS823.003

Added Demand = Constant



TO BE SHOWN: MARGINAL INVESTMENT =

$$\frac{\sum \text{PW OF INVESTMENT CHANGE}}{\sum \text{PW OF DEMAND CHANGE}}$$

= CAPACITY COST = INV/CAP

DEMAND CHANGE

INVESTMENT CHANGE

β = INV = INVESTMENT OF ONE EQUIPMENT UNIT

q = CAP = AVAILABLE CAPACITY OF ONE EQUIPMENT UNIT

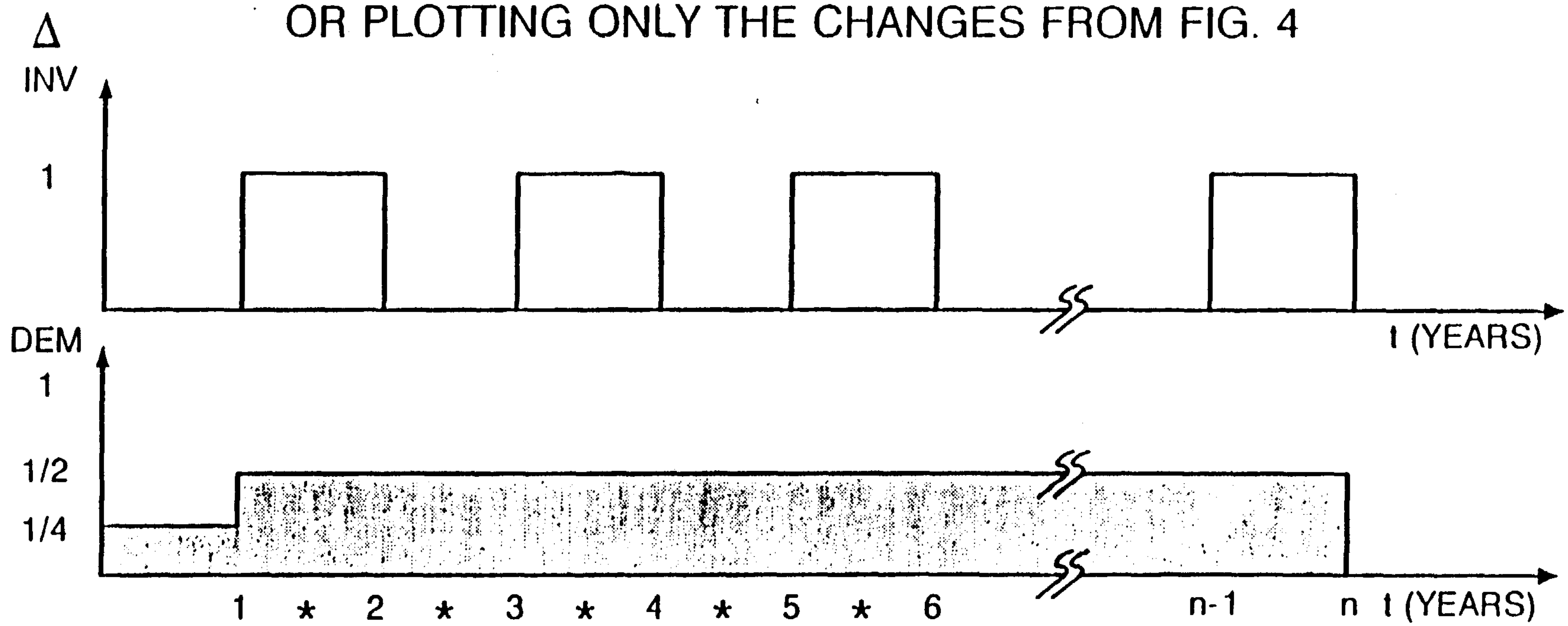
FIGURE 4

LCC90 BVS823.005

$$\text{MARGINAL INVESTMENT} = \frac{\Sigma \text{ PW OF INVESTMENT CHANGE}}{\Sigma \text{ PW OF DEMAND CHANGE}}$$

$$\text{MARGINAL INVESTMENT} = \frac{\Sigma \text{ PW OF } \boxed{\quad\quad}}{\Sigma \text{ PW OF } \boxed{\quad\quad\quad\quad}}$$

OR PLOTTING ONLY THE CHANGES FROM FIG. 4



* USE OF MID YEAR CONVENTION. (FIRST YEAR DEMAND HAS AN EQUAL CHANCE OF ARRIVING IN ANY MONTH, THEREFORE .5 PROB. USED FOR WHOLE YEAR)

FIGURE 4A

LCC90 BVS823.004

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	ORIG DE-MAND * CAP	ORIG IN-VEST-MENT * INV	NEW DE-MAND * CAP	NEW IN-VEST-MENT * INV	CHNGE IN DEMND * CAP	CHNGE IN INVST-MENT * INV	PW CHNGE IN DEMND * CAP	PW CHNGE IN INVST-MENT * INV	MID YR	MID YR CUM MONTH	MONTH-LY INTRST FRACT @ 12%	CUM PW CHNGE IN DEMND * CAP	CUM PW CHNGE IN INVST-MENT * INV	(CUM PW CHG IN INVEST / CUM PW CHG IN DEMAND) * (INV / CAP)
	FIG 4	FIG 4	FIG 4	FIG 4	(D-B)* CAP	(E-C)* INV	F / (1+L)^K * CAP	G / (1+L)^K * INV	HALF YEAR POINT	J * 12	12% / 100* 12)	CUM SUM OF H	CUM SUM OF I	(N / M) * (INV / CAP)
1	0.50	1	0.75	1	0.25		0.23551	0	0.5	6	0.01	0.2355	0	0.0000000000
2	1.00	1	1.50	2	0.50	1.00	0.41800	0.83601	1.5	18	0.01	0.6535	0.8360	1.2792529040
3	1.50	2	2.00	2	0.50		0.37096	0	2.5	30	0.01	1.0244	0.8360	0.8160795044
4	2.00	2	2.50	3	0.50	1.00	0.32920	0.65841	3.5	42	0.01	1.3536	1.4944	1.1039715568
5	2.50	3	3.00	3	0.50		0.29215	0	4.5	54	0.01	1.6458	1.4944	0.9080040374
6	3.00	3	3.50	4	0.50	1.00	0.25927	0.51854	5.5	66	0.01	1.9051	2.0129	1.0566173192
7	3.50	4	4.00	4	0.50		0.23009	0	6.5	78	0.01	2.1352	2.0129	0.9427552513
8	4.00	4	4.50	5	0.50	1.00	0.20419	0.40839	7.5	90	0.01	2.3394	2.4213	1.0350369422
9	4.50	5	5.00	5	0.50		0.18121	0	8.5	102	0.01	2.5206	2.4213	0.9606258323
10	5.00	5	5.50	6	0.50	1.00	0.16081	0.32163	9.5	114	0.01	2.6814	2.7430	1.0229615686
11	5.50	6	6.00	6	0.50		0.14271	0	10.5	126	0.01	2.8241	2.7430	0.9712667279
12	6.00	6	6.50	7	0.50	1.00	0.12665	0.25330	11.5	138	0.01	2.9508	2.9963	1.0154218988
13	6.50	7	7.00	7	0.50		0.11239	0	12.5	150	0.01	3.0632	2.9963	0.9781627096
14	7.00	7	7.50	8	0.50	1.00	0.09974	0.19949	13.5	162	0.01	3.1629	3.1958	1.0103878924
15	7.50	8	8.00	8	0.50		0.08852	0	14.5	174	0.01	3.2514	3.1958	0.9828799792
16	8.00	8	8.50	9	0.50	1.00	0.07855	0.15711	15.5	186	0.01	3.3300	3.3529	1.0068747739
17	8.50	9	9.00	9	0.50		0.06971	0	16.5	198	0.01	3.3997	3.3529	0.9862273724
18	9.00	9	9.50	10	0.50	1.00	0.06187	0.12374	17.5	210	0.01	3.4616	3.4766	1.0043466803
19	9.50	10	10.00	10	0.50		0.05490	0	18.5	222	0.01	3.5165	3.4766	0.9886649601
20	10.00	10	10.50	11	0.50	1.00	0.04872	0.09745	19.5	234	0.01	3.5652	3.5741	1.0024869996
21	10.50	11	11.00	11	0.50		0.04324	0	20.5	246	0.01	3.6085	3.5741	0.9904736664
22	11.00	11	11.50	12	0.50	1.00	0.03837	0.07675	21.5	258	0.01	3.6468	3.6508	1.0010967770
23	11.50	12	12.00	12	0.50		0.03405	0	22.5	270	0.01	3.6809	3.6508	0.9918345209
24	12.00	12	12.50	13	0.50	1.00	0.03022	0.06044	23.5	282	0.01	3.7111	3.7113	1.0000449286
25	12.50	13	13.00	13	0.50		0.02682	0	24.5	294	0.01	3.7379	3.7113	0.9928691598
26	13.00	13	13.50	14	0.50	1.00	0.02380	0.04760	25.5	306	0.01	3.7617	3.7589	0.9992418321
27	13.50	14	14.00	14	0.50		0.02112	0	26.5	318	0.01	3.7829	3.7589	0.9936620414
28	14.00	14	14.50	15	0.50	1.00	0.01874	0.03749	27.5	330	0.01	3.8016	3.7964	0.9986243963
29	14.50	15	15.00	15	0.50		0.01663	0	28.5	342	0.01	3.8182	3.7964	0.9942733543
30	15.00	15	15.50	16	0.50	1.00	0.01476	0.02952	29.5	354	0.01	3.8330	3.8259	0.9981471664
31	15.50	16	16.00	16	0.50		0.01310	0	30.5	366	0.01	3.8461	3.8259	0.9947468862
32	16.00	16	16.50	17	0.50	1.00	0.01162	0.02325	31.5	378	0.01	3.8577	3.8492	0.9977767848
33	16.50	17	17.00	17	0.50		0.01031	0	32.5	390	0.01	3.8681	3.8492	0.9951150213
34	17.00	17	17.50	18	0.50	1.00	0.00915	0.01831	33.5	402	0.01	3.8772	3.8675	0.9974884107
35	17.50	18	18.00	18	0.50		0.00812	0	34.5	414	0.01	3.8853	3.8675	0.9954020246
36	18.00	18	18.50	19	0.50	1.00	0.00721	0.01442	35.5	426	0.01	3.8926	3.8819	0.9972633282

MARGINAL COST = SUM OF THE PRESENT WORTH OF INVESTMENT CHANGES / SUM OF THE PRESENT WORTH OF DEMAND CHANGES

= INV / CAP * column O

PW CAP MARGINAL = CAPACITY COST

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FIGURE 4B (AT A 12 % DISCOUNT RATE)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
YEAR	ORIG DE-MAND * CAP	ORIG IN-VEST-MENT * INV	NEW DE-MAND * CAP	NEW IN-VEST-MENT * INV	CHNGE IN DEMND * CAP	CHNGE IN INVST-MENT * INV	PW CHNGE IN DEMND * CAP	PW CHNGE IN INVST-MENT * INV	MID YR	MID YR CUM MONTH	MONTH-LY INTRST FRACT @ 6%	CUM PW CHNGE IN DEMND * CAP	CUM PW CHNGE IN INVST-MENT * INV	(CUM PW CHG IN INVEST / CUM PW CHG IN DEMAND) * (INV / CAP)
	FIG 4	FIG 4	FIG 4	FIG 4	(D-B)* CAP	(E-C)* INV	F / (1+L)^K * CAP	G / (1+L)^K * INV	HALF YEAR POINT	J * 12	6% / (100* 12)	CUM SUM OF H	CUM SUM OF I	(N / M) * (INV / CAP)
1	0.50	1	0.75	1	0.25		0.24262	0	0.5	6	0.005	0.2426	0	0.0000000000
2	1.00	1	1.50	2	0.50	1.00	0.45706	0.91412	1.5	18	0.005	0.6996	0.9141	1.3064731977
3	1.50	2	2.00	2	0.50		0.43051	0	2.5	30	0.005	1.1302	0.9141	0.8088175777
4	2.00	2	2.50	3	0.50	1.00	0.40550	0.81100	3.5	42	0.005	1.5357	1.7251	1.1233482346
5	2.50	3	3.00	3	0.50		0.38194	0	4.5	54	0.005	1.9176	1.7251	0.9996076790
6	3.00	3	3.50	4	0.50	1.00	0.35975	0.71951	5.5	66	0.005	2.2774	2.4446	1.0734334558
7	3.50	4	4.00	4	0.50		0.33885	0	6.5	78	0.005	2.6162	2.4446	0.9344035115
8	4.00	4	4.50	5	0.50	1.00	0.31917	0.63834	7.5	90	0.005	2.9354	3.0830	1.0502659084
9	4.50	5	5.00	5	0.50		0.30062	0	8.5	102	0.005	3.2360	3.0830	0.9526969311
10	5.00	5	5.50	6	0.50	1.00	0.28316	0.56632	9.5	114	0.005	3.5192	3.6493	1.0369647810
11	5.50	6	6.00	6	0.50		0.26671	0	10.5	126	0.005	3.7859	3.6493	0.9639124019
12	6.00	6	6.50	7	0.50	1.00	0.25121	0.50243	11.5	138	0.005	4.0371	4.1517	1.0283845345
13	6.50	7	7.00	7	0.50		0.23662	0	12.5	150	0.005	4.2738	4.1517	0.9714465946
14	7.00	7	7.50	8	0.50	1.00	0.22287	0.44575	13.5	162	0.005	4.4966	4.5975	1.0224269314
15	7.50	8	8.00	8	0.50		0.20993	0	14.5	174	0.005	4.7066	4.5975	0.9768233305
16	8.00	8	8.50	9	0.50	1.00	0.19773	0.39546	15.5	186	0.005	4.9043	4.9929	1.0180759939
17	8.50	9	9.00	9	0.50		0.18624	0	16.5	198	0.005	5.0905	4.9929	0.9808281164
18	9.00	9	9.50	10	0.50	1.00	0.17542	0.35085	17.5	210	0.005	5.2660	5.3438	1.0147798441
19	9.50	10	10.00	10	0.50		0.16523	0	18.5	222	0.005	5.4312	5.3438	0.9839070610
20	10.00	10	10.50	11	0.50	1.00	0.15563	0.31127	19.5	234	0.005	5.5868	5.6551	1.0122128066
21	10.50	11	11.00	11	0.50		0.14659	0	20.5	246	0.005	5.7334	5.6551	0.9863323635
22	11.00	11	11.50	12	0.50	1.00	0.13807	0.27615	21.5	258	0.005	5.8715	5.9312	1.0101702443
23	11.50	12	12.00	12	0.50		0.13005	0	22.5	270	0.005	6.0016	5.9312	0.9882795700
24	12.00	12	12.50	13	0.50	1.00	0.12250	0.24500	23.5	282	0.005	6.1241	6.1762	1.0085170795
25	12.50	13	13.00	13	0.50		0.11538	0	24.5	294	0.005	6.2395	6.1762	0.9898670015
26	13.00	13	13.50	14	0.50	1.00	0.10868	0.21736	25.5	306	0.005	6.3481	6.3936	1.0071605329
27	13.50	14	14.00	14	0.50		0.10236	0	26.5	318	0.005	6.4505	6.3936	0.9911773324
28	14.00	14	14.50	15	0.50	1.00	0.09642	0.19284	27.5	330	0.005	6.5469	6.5864	1.0060347560
29	14.50	15	15.00	15	0.50		0.09081	0	28.5	342	0.005	6.6377	6.5864	0.9922700851
30	15.00	15	15.50	16	0.50	1.00	0.08554	0.17108	29.5	354	0.005	6.7233	6.7575	1.0050917126
31	15.50	16	16.00	16	0.50		0.08057	0	30.5	366	0.005	6.8039	6.7575	0.9931892108
32	16.00	16	16.50	17	0.50	1.00	0.07589	0.15178	31.5	378	0.005	6.8798	6.9093	1.0042955332
33	16.50	17	17.00	17	0.50		0.07148	0	32.5	390	0.005	6.9512	6.9093	0.9939678711
34	17.00	17	17.50	18	0.50	1.00	0.06733	0.13466	33.5	402	0.005	7.0186	7.0440	1.0036188915
35	17.50	18	18.00	18	0.50		0.06341	0	34.5	414	0.005	7.0820	7.0440	0.9946315576
36	18.00	18	18.50	19	0.50	1.00	0.05973	0.11946	35.5	426	0.005	7.1417	7.1634	1.0030406046

MARGINAL COST = SUM OF THE PRESENT WORTH OF INVESTMENT CHANGES / SUM OF THE PRESENT WORTH OF DEMAND CHANGES

= INV / CAP * column O

= INV / CAP * (APPROX 1.0) = CAPACITY COST

FIGURE 4C (AT A 6 % DISCOUNT RATE)